



Problem 2. «Simple ideas for primes»

Problem for a special prize!

It is well known that prime numbers form a very special and mysterious class. They have too many applications in public-key cryptography (and not only there).

During many years (to be precise, hundreds of years) mathematicians think about simple constructions for prime numbers. Let us consider particular examples in this area.

- *Fermat numbers*, $F_k = 2^{2^k} + 1$, where integer k starts from 0, give us five prime numbers: $F_0 = 3$, $F_1 = 5$, $F_2 = 17$, $F_3 = 257$, $F_4 = 65\,537$. However, the next number, $F_5 = 4\,284\,967\,297 = 641 \cdot 6\,700\,417$, is composite as was proven latter. So far no more prime Fermat numbers were found.

- Several *Mersenne numbers*, $M_k = 2^k - 1$, are prime. For example, $M_2 = 3$, $M_3 = 7$, $M_5 = 31$, $M_7 = 127$, while M_{11} is already composite. Here we consider Mersenne numbers with k being prime, since it is a necessary condition for M_k to be prime. Up to now there are known 51 prime Mersenne numbers. The last one found prime Mersenne number is $M_{82\,589\,933}$; it was obtained in 2018 and up to now it is the biggest known prime number.

- From time to time, some original ideas appear. For instance, seven consecutive (by construction) numbers 31, 331, 3 331, 33 331, 333 331, 3 333 331 and 33 333 331 are prime! But the next number 333 333 331 is composite, since it can be divided by 17.

Let us say that Fermat prime numbers have the *sequence primality parameter* equal to 5, Mersenne prime numbers have it equal to 4, while for the last construction this parameter equals 7. So, the sequence primality parameter stands for the length of the longest subsequence of prime numbers in the sequence of numbers constructed.

Propose your own construction of integer numbers with the sequence primality parameter as big as possible. There is an important condition: every number from your construction should be presented explicitly (as in the examples above), i.e. the number should not depend on the previous numbers but should depend only on its index in the sequence.

