Problem 2. «Interpolation with Errors»

Let $n = 2022$ and let $\mathbb{Z}_n$ be the ring of integers modulo n. Given $x_i, y_i \in \mathbb{Z}_n$ for $i \in \{1, \ldots, 324\}$, find monic polynomials

$$f(x) = x^{16} + \alpha_{15}x^{15} + \ldots + \alpha_1 x + \alpha_0,$$
$$g(x) = x^{16} + \beta_{15}x^{15} + \ldots + \beta_1 x + \beta_0$$

of degree $d = 16$ and coefficients from $\mathbb{Z}_n$ such that the relation

$$y_i = \frac{f(x_i)}{g(x_i)} = \frac{x_i^{16} + \alpha_{15}x_i^{15} + \ldots + \alpha_1 x_i + \alpha_0}{x_i^{16} + \beta_{15}x_i^{15} + \ldots + \beta_1 x_i + \beta_0}$$

holds for at least 90 of the indices $i \in \{1, \ldots, 324\}$.

Note. The coefficients $\beta_0, \ldots, \beta_{15}$ are such that the denominator of the above fraction is invertible for all possible values of $x_i \in \mathbb{Z}_n$. It can be assumed that they are sampled uniformly at random from all such sets of values. Furthermore, the positions and error values can be also assumed to be sampled uniformly at random.

The attachment contains a CSV file with 324 triplets (i, x_i, y_i).