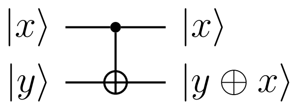
Problem 5. «Reversing a gate»

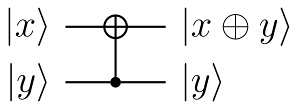
Daniel continues to study quantum circuits. A controlled NOT (CNOT) gate is the most complex quantum gate from the universal set of gates required for quantum computation. This gate acts on two qubits and makes the following transformation:

$$|00\rangle \to |00\rangle\,, \quad |01\rangle \to |01\rangle\,, \quad |10\rangle \to |11\rangle\,, \quad |11\rangle \to |10\rangle\,.$$

This gate is clearly asymmetric. The first qubit is considered as control one, and the second is as a target one. CNOT is described by the following quantum circuit ($x, y \in \mathbb{F}_2$):



The problem. Help Daniel to design a circuit in a special way that reverses CNOT gate:



It makes the following procedure: $|00\rangle \to |00\rangle\,,\ |01\rangle \to |11\rangle\,,\ |10\rangle \to |10\rangle\,,\ |11\rangle \to |01\rangle$. To do this you should modify the original CNOT gate without re-ordering the qubits but via adding some single-qubit gates instead from the following ones:

Pauli-X gate	$\vert x\rangle$ — $\boxed{X}$ — $\vert x \oplus 1\rangle$	acts on a single qubit in the state $\vert x\rangle$, $x \in \{0,1\}$
Pauli-Z gate	$\vert x\rangle$ — $\boxed{Z}$ — $(-1)^x \vert x\rangle$	acts on a single qubit in the state $\vert x\rangle$, $x \in \{0,1\}$
Hadamard gate	$\vert x\rangle$ — $\boxed{H}$ — $\frac{\vert 0\rangle + (-1)^x \vert 1\rangle}{\sqrt{2}}$	acts on a single qubit in the state $\vert x\rangle$, $x \in \{0,1\}$

Remark. Let us briefly formulate the key points of quantum circuits. A qubit is a two-level quantum mechanical system whose state $|\psi\rangle$ is the superposition of basis quantum states $|0\rangle$ and $|1\rangle$. The superposition is written as $|\psi\rangle = \alpha_0 |0\rangle + \alpha_1 |1\rangle$, where α_0 and α_1 are complex numbers, called amplitudes, that possess $|\alpha_0|^2 + |\alpha_1|^2 = 1$. The amplitudes α_0 and α_1 have the following physical meaning: after the measurement of a qubit which has the state $|\psi\rangle$, it will be found in the state $|0\rangle$ with probability $|\alpha_0|^2$ and in the state $|1\rangle$ with probability $|\alpha_1|^2$. In order to operate with multi-qubit systems, we consider the bilinear operation $\otimes : |x\rangle, |y\rangle \to |x\rangle \otimes |y\rangle$ on $x, y \in \{0,1\}$ which is defined on pairs $|x\rangle, |y\rangle$, and by bilinearity is expanded on the space of all linear combinations of $|0\rangle$ and $|1\rangle$. When we have two qubits in states $|\psi\rangle$ and $|\varphi\rangle$ correspondingly, the state of the whole system of these two qubits is $|\psi\rangle \otimes |\varphi\rangle$. In general, for two qubits we have $|\psi\rangle = \alpha_{00}|0\rangle \otimes |0\rangle + \alpha_{01}|0\rangle \otimes |1\rangle + \alpha_{10}|1\rangle \otimes |0\rangle + \alpha_{11}|1\rangle \otimes |1\rangle$. The physical meaning of complex numbers α_{ij} is the same as for one qubit, so we have the essential restriction $|\alpha_{00}|^2 + |\alpha_{01}|^2 + |\alpha_{10}|^2 + |\alpha_{11}|^2 = 1$. We use more brief notation $|a\rangle \otimes |b\rangle \equiv |ab\rangle$. In order to verify your circuits, you can use different quantum circuit simulators, for example `https://algassert.com/quirk`.