Problem 11. «Distance to affine functions»

Given two functions F and G from $\mathbb{F}_2^n$ (or $\mathbb{F}_{2^n}$) to itself, their Hamming distance equals by definition the number of inputs x at which $F(x) \neq G(x)$. The minimum Hamming distance between any such function F and all affine functions A is known to be strictly smaller than $2^n - n - 1$ if $n \geqslant 4$.

Consider the following problems. Each of them is a **Problem for a special prize!**

Q1 Find a better upper bound valid for every n.

Q2 If **Q1** is unsuccessful, find constructions of infinite classes of functions F having a distance to affine functions as large as possible (infinite classes meaning that these functions are in numbers of variables ranging in an infinite set, such as all positive integers, possibly of some parity for instance).

Q3 If **Q1** and **Q2** are unsuccessful, find constructions (possibly with a computer; then a representation of these functions will be needed, such as their algebraic normal form or their univariate representation) of functions F in fixed numbers of variables having a distance to affine functions as large as possible.

Remark. We recall that an affine function A is a function satisfying $A(x)+A(y)+A(z) = A(x+y+z)$ for all inputs x, y, z.