



Problem 6. «Two strings»

Carol takes inspiration from different strings and comes up with unusual ways to build them. Today, she starts with a binary string A_n constructed by induction in the following way. Let $A_1 = 0$ and $A_2 = 1$. For $n > 2$, the string A_n is defined by concatenating the strings A_{n-1} and A_{n-2} from left to right, i. e. $A_n = A_{n-1}A_{n-2}$.

Together with A_n consisting of “0”s and “1”s, Carol constructs a ternary string B_n consisting of “-1”s, “0”s and “1”s. Let $A_n = a_1 \dots a_m$ for appropriate m , where $a_i \in \{0, 1\}$; then $B_n = b_1 \dots b_\ell$, where $\ell = \lceil m/2 \rceil$ and $b_i \in \{-1, 0, 1\}$ is defined as follows:

$$b_i = a_{2i-1} - a_{2i} \text{ for } i = 1, \dots, \ell \text{ (the exceptional case } b_\ell = a_m \text{ if } m \text{ is odd).}$$

Help Carol to find all n such that B_n has the same number of “1”s and “-1”s.

Example. The strings A_n and B_n for small n are the following:

$$\begin{aligned} A_3 &= A_2A_1 = 10, & A_4 &= A_3A_2 = 101, & A_5 &= A_4A_3 = 10110, & A_6 &= A_5A_4 = 10110101. \\ B_3 &= 1, & B_4 &= 11, & B_5 &= 100, & B_6 &= 10(-1)(-1). \end{aligned}$$

