Problem 4. «Related passwords»

Tim and Ann want to create curiously related passwords for their cryptosystem. A password is a 9-digit decimal number. To start, they choose a random number $e_1e_2...e_9$ that has nine (not necessarily distinct) decimal digits.

- Tim finds a password $d_1d_2...d_9$ such that each of the numbers formed by replacing just one of the digits d_i in $d_1d_2...d_9$ by the corresponding digit e_i is divisible by 7.

- Ann finds a password $f_1f_2...f_9$ in similar but not the same way: each of the nine numbers formed by replacing one of the e_i in $e_1e_2...e_9$ by f_i is divisible by 7.

Show that for each i, $d_i - f_i$ is divisible by 7 for any of Tim's and Ann's passwords!

Example. Let $e_1e_2...e_9 = 448259545$. Then Tim's password can be $d_1d_2...d_9 = 199501996$ and Ann's password can be $f_1f_2...f_9 = 822571226$.