Problem 7. «CPA game»

Suppose we have a system for the encryption of binary messages. The system has the following characteristics:

- Every message is divided into blocks of length n that are called plaintexts (it is supposed that the length of messages is divisible by n).
- The system employs a block cipher with the encryption function E in cipher block chaining (CBC) mode (see the picture below). A block, an initialization vector IV and a key lengths are equal to n. The result of encryption of the message is a concatenation of IV and the ciphertexts of all plaintexts it consists of.
- The IV for the first message is chosen randomly by using a secure pseudorandom number generator. The last ciphertext block of the i-th message is used as the IV for the $(i+1)$-st message.

Let Alice be an honest user of the system. Victor, an adversary, convinced her to play **chosen–plaintext attack game** (CPA game) with him.

The game is the following:

1. Alice selects a key $k \in \{0,1\}^n$ and chooses a bit $b \in \{0,1\}$.

2. Victor submits a sequence of q queries to Alice. For $i = 1, 2, \ldots, q$ repeat

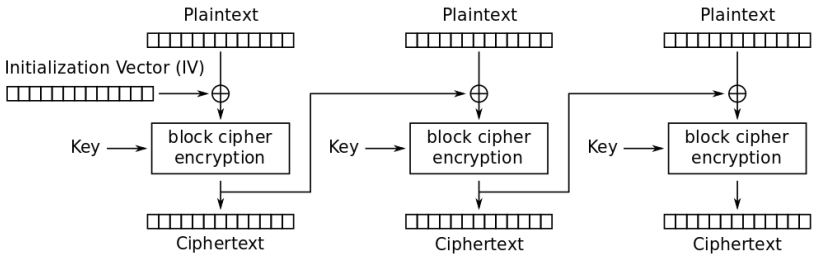
(a) Victor chooses a pair of messages, $m_{i,0}, m_{i,1}$ of the same length.

(b) Alice encrypts $m_{i,b}$ with the key k and gets c_i (the sequence of corresponding IV and ciphertexts). She sends c_i to Victor.

3. Victor outputs a bit $b^* \in \{0,1\}$.

Let W be the event that Victor guesses the bit, that is $b^* = b$. We define Victors's advantage with respect to E as CPAadv $:= |\Pr[\mathrm{W}] - 1/2|$. Victor wins the game if he can build an efficient algorithm such that CPAadv is not negligible.

Task. Construct an efficient probabilistic polynomial-time (PPT) algorithm that wins the CPA game against this implementation with an advantage close to $1/2$.



Cipher Block Chaining (CBC) mode encryption