



Problem 11. «Disjunct Matrices»

Special Prize from the Program Committee!

Disjunct Matrices are used in some key distribution protocols for traitor tracing. Disjunct Matrices (DM) are a particular kind of binary matrices which have been especially applied to solve the Non-Adaptive Group Testing (NAGT) problem, where the task is to detect any configuration of t defectives out of a population of N items. Traditionally, the methods used to construct DM leverage on error-correcting codes and other related algebraic techniques.

Let $A = (x_1^\top, x_2^\top, \dots, x_N^\top)$ be an $M \times N$ binary matrix. Then, A is called t -disjunct if, for all subsets of t columns $S = \{x_{i_1}, \dots, x_{i_t}\}$, and for all remaining columns $x_j \notin S$, it holds that

$$\text{supp}(x_j) \not\subseteq \bigcup_{k=1}^t \text{supp}(x_{i_k}),$$

where $\text{supp}(x)$ denotes the set of coordinate positions of a binary vector x with 1s.

In other words, a matrix A is t -disjunct if for every subset S of t columns the support of any other column is not contained in the union of the supports of the columns in S .

Prove what is the minimum number of rows in a 5-disjunct matrix.