Problem 5. «Solutions of the equation»

Alice is studying special functions that are used in symmetric ciphers. Let E^n be the set of all binary vectors $x = (x_1, x_2, \ldots, x_n)$ of length n, where x_i is either 0 or 1. Given two vectors x and y from E^n consider their sum $x \oplus y = (x_1 \oplus y_1, \ldots, x_n \oplus y_n)$, where $\oplus$ is addition modulo 2.

Example. If $n = 3$, then $E^3 = \{(000), (001), (010), (011), (100), (101), (110), (111)\}$. Let $x = (010)$ and $y = (011)$, then vector $x \oplus y$ is equal to $(010) \oplus (011) = (0 \oplus 0, 1 \oplus 1, 0 \oplus 1) = (001)$.

We will say that a function F maps E^n to E^n if it transforms any vector x from E^n into some vector $F(x)$ from E^n.

Example. Let $n = 2$. For instance, we can define F that maps E^2 to E^2 as follows: $F(00) = (00)$, $F(01) = (10)$, $F(10) = (11)$ and $F(11) = (10)$.

Alice found a function S that maps E^6 to E^6 in such a way that the vectors $S(x)$ and $S(y)$ are not equal for any nonequal vectors x and y. Also, S has another curious property: the equation

$$S(x) \oplus S(x \oplus a) = b$$

has either 0 or 2 solutions for any nonzero vector a from E^6 and any vector b from E^6.

Find the number of pairs (a, b) such that this equation has exactly **2** solutions!