Problem 3. «Key matrices»

Let n be an **odd** positive integer. In some cipher, a key is a binary $n \times n$ matrix

$$A = \begin{pmatrix} a_{1,1} & a_{1,2} & \dots & a_{1,n} \\ a_{2,1} & a_{2,2} & \dots & a_{2,n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{n,1} & a_{n,2} & \dots & a_{n,n} \end{pmatrix},$$

where $a_{i,j}$ is either 0 or 1, such that each diagonal of any length $1, 2, \dots, n-1, n$ contains an **odd** number of 1s.

What is the minimal and the maximal number of 1s that can be placed in a key matrix A?

Example. For $n = 3$, diagonals are the following ten lines:

$$A = \begin{pmatrix} a_{1,1} & a_{1,2} & a_{1,3} \\ a_{2,1} & a_{2,2} & a_{2,3} \\ a_{3,1} & a_{3,2} & a_{3,3} \end{pmatrix}$$