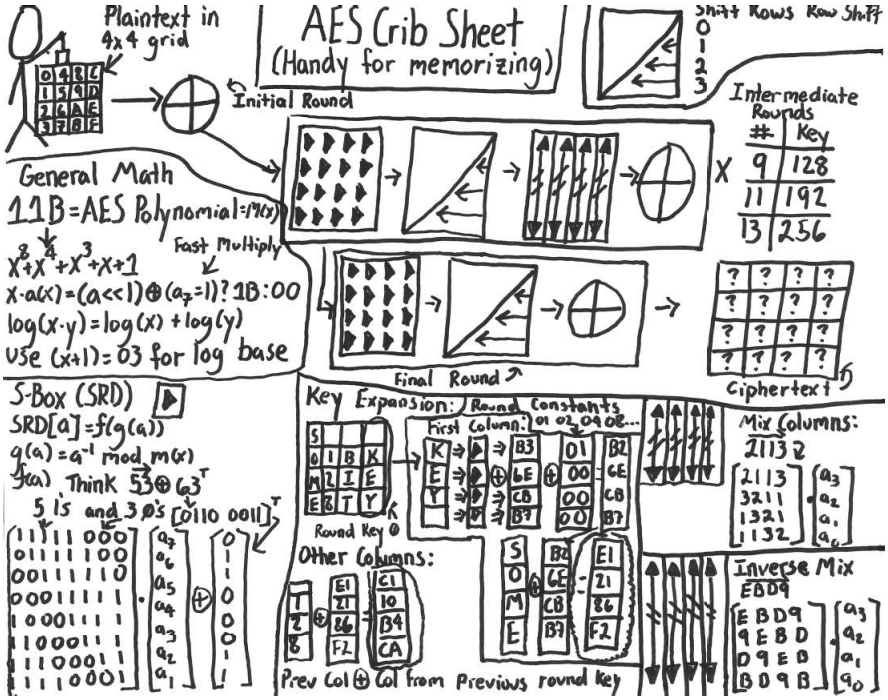
Problem 2. «Zerosum at AES»

Let $\texttt{AES}_0$ be a mapping that represents the algorithm AES-256 with the all-zero key. Let $X_1, \ldots, X_{128} \in \mathbb{F}_2^{128}$ be pairwise different vectors such that

$$\bigoplus_{i=1}^{128} X_i = \bigoplus_{i=1}^{128} \texttt{AES}_0(X_i).$$

1. Propose an effective algorithm to find an example of such vectors $X_1, \ldots, X_{128}$.
2. Provide an example of $X_1, \ldots, X_{128}$.



The picture is from the page of Jeff Moser.