Problem 12. «Big Fermat numbers»

Special Prize from the Program Committee!

It is known that constructing big prime numbers is very actual and complicated problem interesting for cryptographic applications. One of the popular way to find them is... to guess! For example to guess them between numbers of some special form. For checking there are Mersenne numbers $2^k - 1$, Fermat numbers $F_k = 2^{2^k} + 1$ for nonnegative integer k, etc.

Let us concentrate our attention on Fermat's numbers.

It is known that Fermat numbers $F_0 = 3$, $F_1 = 5$, $F_2 = 17$, $F_3 = 257$, $F_4 = 65537$ are prime. But the number $F_5 = 4\,284\,967\,297 = 641 \cdot 6\,700\,417$ is already composite as was proven by L. Euler in XVIII.

For now it is known that all Fermat numbers, where $k = 5, \ldots, 32$, are composite and there is the hypothesis that every Fermat number F_k, where $k \geqslant 5$ is composite.

Could you prove that for any big number N there exists a composite Fermat number F_k such that $F_K > N$?

381	382	383	384	385	386	387	388	389	390	391	392	393	394	395	396	397	398	399	400	401
380	307	308	309	310	311	312	313	314	315	316	317	318	319	320	321	322	323	324	325	402
379	306	241	242	243	244	245	246	247	248	249	250	251	252	253	254	255	256	257	326	403
378	305	240	183	184	185	186	187	188	189	190	191	192	193	194	195	196	197	258	327	404
377	304	239	182	133	134	135	136	137	138	139	140	141	142	143	144	145	198	259	328	405
376	303	238	181	132	91	92	93	94	95	96	97	98	99	100	101	146	199	260	329	406
375	302	237	180	131	90	57	58	59	60	61	62	63	64	65	102	147	200	261	330	407
374	301	236	179	130	89	56	31	32	33	34	35	36	37	66	103	148	201	262	331	408
373	300	235	178	129	88	55	30	13	14	15	16	17	38	67	104	149	202	263	332	409
372	299	234	177	128	87	54	29	12	3	4	5	18	39	68	105	150	203	264	333	410
371	298	233	176	127	86	53	28	11	2	1	6	19	40	69	106	151	204	265	334	411
370	297	232	175	126	85	52	27	10	9	8	7	20	41	70	107	152	205	266	335	412
369	296	231	174	125	84	51	26	25	24	23	22	21	42	71	108	153	206	267	336	413
368	295	230	173	124	83	50	49	48	47	46	45	44	43	72	109	154	207	268	337	414
367	294	229	172	123	82	81	80	79	78	77	76	75	74	73	110	155	208	269	338	415
366	293	228	171	122	121	120	119	118	117	116	115	114	113	112	111	156	209	270	339	416
365	292	227	170	169	168	167	166	165	164	163	162	161	160	159	158	157	210	271	340	417
364	291	226	225	224	223	222	221	220	219	218	217	216	215	214	213	212	211	272	341	418
363	290	289	288	287	286	285	284	283	282	281	280	279	278	277	276	275	274	273	342	419
362	361	360	359	358	357	356	355	354	353	352	351	350	349	348	347	346	345	344	343	420
441	440	439	438	437	436	435	434	433	432	431	430	429	428	427	426	425	424	423	422	421