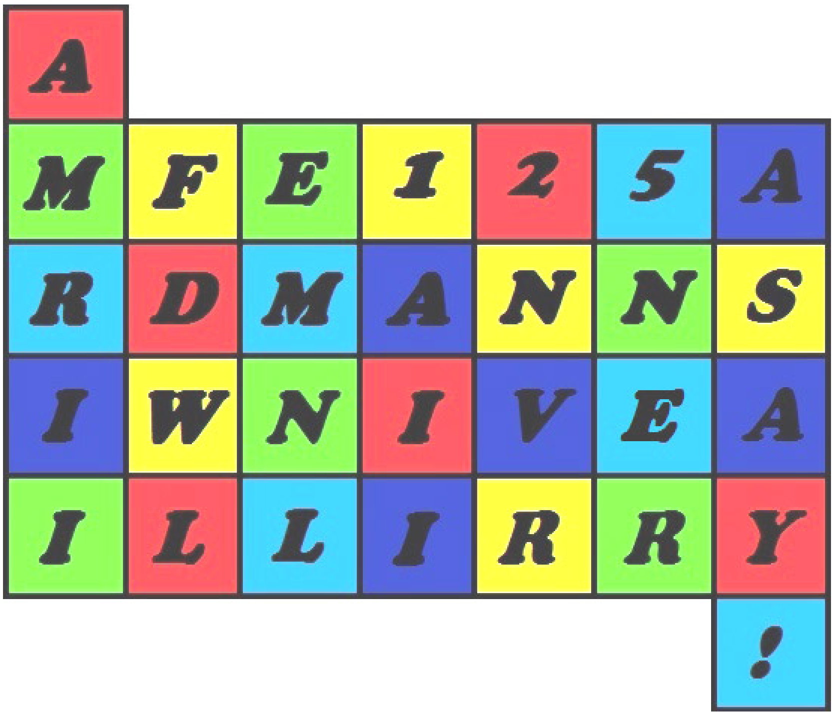
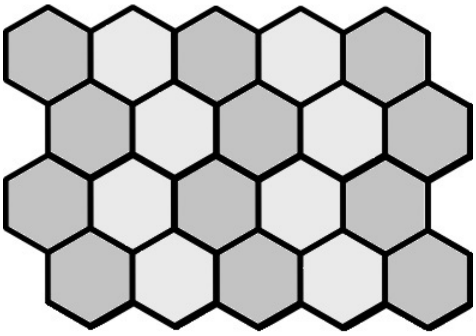
Problem 1. «Cipher from the pieces»

Recover the original message, splitting the figure into equal pieces such that each color occurs once in every piece.



Problem 2. «Get an access»

To get an access to the safe one should put 20 non-negative integers in the following cells. The safe will be opened if and only if the sum of any two numbers is even number k, such that $4 \leqslant k \leqslant 8$, and each possible sum occurs at least once. Find the sum of all these numbers.



Problem 3. «Find the key»

The key of a cipher is the set of positive integers a, b, c, d, e, f, g, such that the following relation holds:

$$a^3 + b^3 + c^3 + d^3 + e^3 + f^3 + g^3 = 2016^{2017}.$$

Find the key!

Problem 4. «Labyrinth»

Read the message hidden in the labyrinth!

Problem 5. «System of equations»

Analyzing a cipher Caroline gets the following system of equations in binary variables $x_1, x_2, \ldots, x_{16} \in \{0,1\}$ that represent the unknown bits of the secrete key:

$$\begin{cases} x_1x_3 \oplus x_2x_4 = x_5 - x_6, \\ x_{14} \oplus x_{11} = x_{12} \oplus x_{13} \oplus x_{14} \oplus x_{15} \oplus x_{16}, \\ (x_8 + x_9 + x_7)^2 = 2(x_6 + x_{11} + x_{10}), \\ x_{13}x_{11} \oplus x_{12}x_{14} = -(x_{16} - x_{15}), \\ x_5x_1x_6 = x_4x_2x_3, \\ x_{11} \oplus x_8 \oplus x_7 = x_{10} \oplus x_6, \\ x_6x_{11}x_{10} \oplus x_7x_9x_8 = 0, \\ \left(\frac{x_{12}+x_{14}+x_{13}}{\sqrt{2}}\right)^2 - x_{15} = x_{16} + x_{11}, \\ x_1 \oplus x_6 = x_5 \oplus x_3 \oplus x_2, \\ x_6x_8 \oplus x_9x_7 = x_{10} - x_{11}, \\ 2(x_5 + x_1 + x_6) = (x_4 + x_3 + x_2)^2, \\ x_{11}x_{13}x_{12} = x_{15}x_{14}x_{16}. \end{cases}$$

Help Caroline to find the all possible keys!

Remark. If you do it in analytic way (without computer calculations) you get twice more scores.

Problem 6. «Biometric pin-code»

Iris is one of the most reliable biometric characteristics of a human. While measuring let us take 16-bit vector from the biometric image of an iris. As in reality, we suppose that two 16-bit biometric images of *the same human* can differ not more than by 10–20%, while biometric images of *different people* have differences at least 40 – 60%.

Let a key k be an arbitrary 5-bit vector. We suppose that the key is a pin-code that should be used in order to get an access to the bank account of a client.

To avoid situation when malefactor can steal the key of a some client and then be able to get an access to his account, the bank decided to combine usage of the key with biometric authentication of a client by iris-code. The following scheme of covering the key with biometric data was proposed:

1) on registration of a client take 16-bit biometric image $b_{template}$ of his iris;

2) extend 5-bit key k to 16-bit string s using Hadamard encoding, i.e. if $k = (k_1, \ldots, k_5)$, where $k_i \in \{0, 1\}$, then s is the vector of values of the Boolean function $f(x_1, \ldots, x_4) = k_1x_1 \oplus \ldots \oplus k_4x_4 \oplus k_5$, where $\oplus$ is summing modulo 2;

3) save the vector $c = b_{template} \oplus s$ on the smart-card and give it to the client. A vector c is called *biometrically encrypted key*.

To get an access to his account a client should

1) take a new 16-bit biometric image b of his iris;

2) using information from the smart-card count 16-bit vector s' as $s' = b \oplus c$;

3) decode s' to the 5-bit vector k' using Hadamard decoding procedure.

Then the bank system checks: if $k' = k$ then the client is authenticated and the key is correct; hence bank provides an access to the account of this client. Otherwise, if $k' \neq k$ then bank signals about an attempt to get illegal access to the bank account.

The problem. Find the 5-bit k of Alice if you know her smart-card data c and a new biometric image b (both are given on the picture).

Remark. Vector of values of a Boolean function f in 4 variables is a binary vector $(f(x^0), f(x^1), \ldots, f(x^{15}))$ of length 16, where $x^0 = (0, 0, 0, 0)$, $x^1 = (0, 0, 0, 1)$, …, $x^{15} = (1, 1, 1, 1)$, ordered by lexicographical order; for, example, vector of values of the function $f(x_1, x_2, x_3, x_4) = x_3 \oplus x_4 \oplus 1$ is equal to (1010101010101010).