Problem 5. «Hypothesis»

Special Prize from the Program Committee!

Prove the following hypothesis or find a counterexample to it.

Hypothesis. For all $n \geqslant 2$ there exists a Boolean function $g : \mathbb{F}_2^{n-1} \to \mathbb{F}_2$ in disjunctive normal form, *where every variable appears not more than one time*, such that a binary sequence $\{u_1, u_2, \ldots\}$ produced for all $t \geqslant 1$ from the initial state $u_1, \ldots, u_n$ by the following rule

$$u_{t+n} = u_t \oplus g(u_{t+1}, u_{t+2}, \ldots, u_{t+n-1})$$

has the maximal possible period equal to 2^n.

Remark I. A Boolean function g in m variables is given in disjunctive normal form if $g(x_1, \ldots, x_m) = A_1 \vee \ldots \vee A_k$, where A_i is a conjunction of variables or their negations, $i = 1, \ldots, k$.

Remark II. In the table, the functions g that confirm the hypothesis for small n are presented.

n	the examples of $g(x_1, \ldots, x_{n-1})$
2	1
3	$x_1 \vee \overline{x}_2$
4	$x_1 \vee \overline{x}_2\overline{x}_3$, $x_1 \vee x_2 \vee \overline{x}_3$
5	$x_2 \vee \overline{x}_1\overline{x}_3\overline{x}_4$, $x_1 \vee x_2x_3 \vee \overline{x}_4$