



Problem 1. «A secret sharing»

Special Prize from the Program Committee!

Alice, Bob and Caroline are going to create a secret sharing system. They choose some subset $M \subseteq \mathbb{F}_2^n$ and want to share a secret element u from M using the following way: the secret is represented as $x \oplus y \oplus z$ where x, y, z are different elements of $\overline{M} = \mathbb{F}_2^n \setminus M$; Alice, Bob and Caroline will store x, y and z correspondingly. Here \mathbb{F}_2^n is the set of all binary vectors of length n .

To use the system, the sets M and \overline{M} should satisfy the following conditions:

- 1) each element $u \in M$ can be represented as $u = x \oplus y \oplus z$, where x, y, z are different elements of \overline{M} ;
- 2) for all different $x, y, z \in \overline{M}$ it is right $x \oplus y \oplus z \in M$.

Help them to implement the system suggesting an explicit construction of the set M for an arbitrary n .