Problem 5. «A binary tape»

A cipher machine works with a binary infinite tape that starts with an input word of length n and all its other elements are zero. The machine encrypts an input word and writes the result instead of it.

The cipher machine can do two operations:

1) copy any symbol of the tape to other position;

2) apply some fixed one-to-one function $S : \mathbb{F}_2^m \to \mathbb{F}_2^m$ to the first m symbols, where $\mathbb{F}_2 = \{0, 1\}$.

Find the conditions for S such that the machine can perform any bijective mapping of words of length n.

Examples of operations.

1) For instance, the machine can copy the third symbol to the fifth place:

1	1	**1**	0	**0**	0	1	1	1	...

the result will be

1	1	1	0	**1**	0	1	1	1	...

2) Let m be 3 and $S(x, y, z) = (x, y, x \oplus z)$; applying S to the first three symbols:

1	**1**	**1**	0	0	0	1	1	1	...

the result will be

1	**1**	**0**	0	0	0	1	1	1	...