

Problem 1. «Key sharing»

A bank safe can be opened with 9 keys inserted in its keyholes in a right order. The keyholes are arranged in a circle. The order of keys is *right* if sum of keys (each key is associated with a natural number) in every three consequent keyholes is divisible by 3.

The safe has two special features: if you insert a key in a keyhole, you can not get it back until all 9 keys are inserted; if the order of inserted 9 keys is wrong, the safe sends «SOS signal» and blocks itself.

Keys were divided between 3 persons: Alice, Bob and Caroline. Together they have a permission to open the bank safe. Their keys are the following:

• Alice: {4,14,24};

• Bob: {34,44,54};

• Caroline: $\{64,74,84\}$.

Today Alice, Bob and Caroline are going to open the safe. But one of them forgot the rule of right order for keys and has already inserted two his keys into consequent keyholes when was stopped by his friends. Prove that Alice, Bob and Caroline still are able to open the safe in this situation.

