Task 1. «A hidden message»

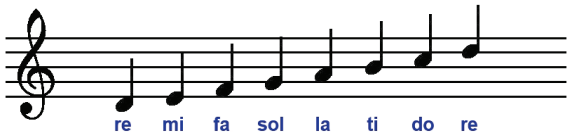
CrYPtogRapHY iS a ScIEnce Of «seCrET wriTinG». FOr aT Least Two THoUsANd yeaRS ThErE haVE bEeN peOPlE WHo WAnTeD to SEnd MESsaGes WHiCh coUlD oNly bEen rEAd bY tHe pEOPLe FoR whOm tHey were iNteNdeD. a loT oF different MEtHODs FoR coNcEalING mEssageS WerE invENtED stARTING WIth AnCIeNt cIPHerS lIKE «SkytaLE» and «ATBAsH» and ending wiTH MOdErn SymmeTRiC ANd PubliC-kEy enCRYptioN ALGOriTHmS SUch aS AeS and Rsa. the dEVELopMENT Of crYPtOgRaPHy cOntiNueS And NEVER sTopS! decrYPt THe mESsaGe tHat iS hIDdEn in thE teXT oF this TASk! tHE aLphabet FoR THE mEssAGE ConsisTs of ALl tWEnTy six enGliSh letTERS from «a» To «z» ANd Six puNCTuaTIoN MARkS « », «.», «,», «!», «?», «'».

Task 2. «A crypto room»

You are in a crypto room with a secret message in hands. Decrypt it!

Task 3. «The musical notation»

Alice and Bob invented a new way for encrypting messages based on musical notations of melodies. They are not very good in musical notations but they know the basic notes «do», «re», «mi», «fa», «sol», «la», «ti», and their places in the staff:



To encrypt a message of length n in English alphabet Alice chooses a melody consisting of n notes. She writes a message under the musical notation of the melody in such a way that each letter of the message corresponds to exactly one note's position in the musical notation. Then for each note («do», «re», ..., «ti») Alice forms the ordered group of corresponding letters. Further she takes a random integer number k_i, $i = 1, \ldots, 7$, and cyclically shifts letters in the i-th group on k_i positions to the right. After that Alice forms the ciphertext by writing letters of the shifted groups under the musical notation again.

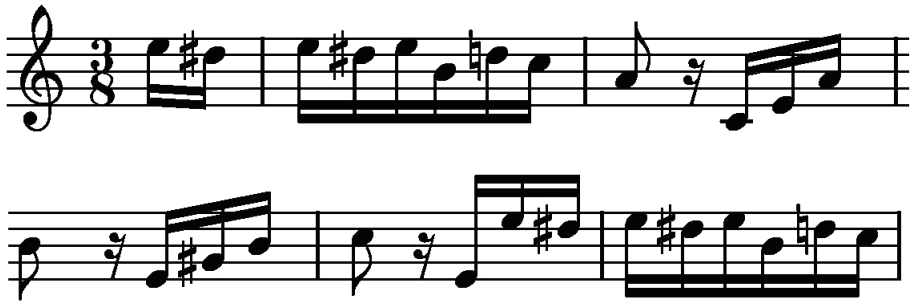
An example. Suppose that Alice wants to send the message `H E L L O`.

The group for «re» is (`E`, `L`); for «mi» — (`H`, `L`, `O`). Alice takes random numbers 2 and 1 for «re» and «mi» respectively. After shifting she gets groups (`E`, `L`) and (`O`, `H`, `L`).
Hence the ciphertext for the message is `O E H L L`.

Decrypt the following ciphertext sent to Bob by Alice:

`R O L E L I S E O E E E H T O M V C P B D E F S O N`

It is known that Alice used the musical notation below.



Task 4. «Boolean cubes»

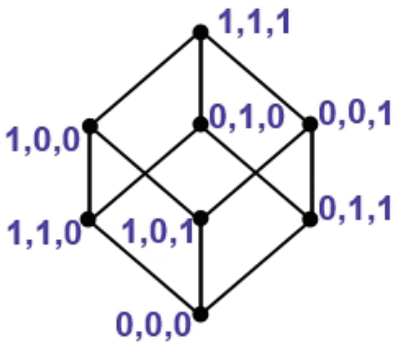
Alice has two cubes E_1 and E_2 of dimension 3 (see the picture below). Their vertices have labels consisting of three integers; for example, (1,0,1) consists of integers 1, 0, 1. Consider an operation A that can be applied for a cube. The operation A contains three steps:

Step 1. Take an arbitrary edge of the cube;

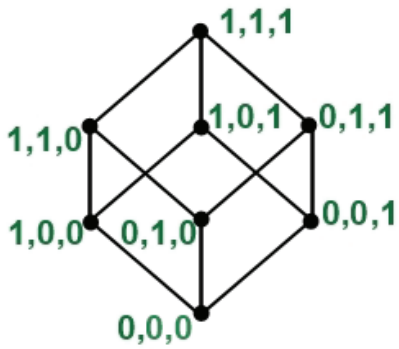
Step 2. Take the number a equal to 1 or -1;

Step 3. Add a to an arbitrary position of the first vertex of the chosen edge. Add a to an arbitrary position of the second vertex of the edge.

Is it possible to get the cube E_2 from the cube E_1 by applying the operation A as many times as necessary? Give your arguments.

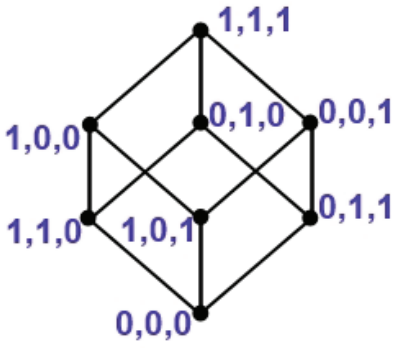


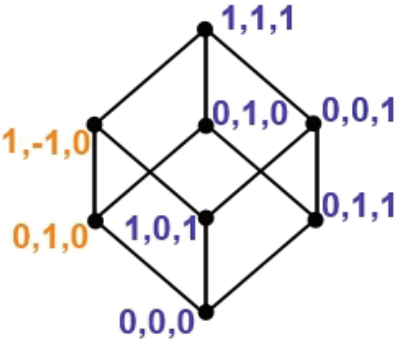
The cube E_1



The cube E_2

An example of applying an operation. Step 1. Take the edge $((1,0,0);(1,1,0))$. Step 2. Let $a = -1$. Step 3. For the vertex $(1,0,0)$ we choose position 2 and for the vertex $(1,1,0)$ we choose position 1; after adding the edge $((1,0,0);(1,1,0))$ becomes $((1,-1,0);(0,1,0))$.





Task 5. «A broken cipher machine»

Mary works on a cipher machine that encrypts messages like this:

Step 1. It represents a message as the natural number $n = \overline{abcdef\ldots}$;

Step 2. Then it sums all the digits in the number, $S_n = a+b+c+d+e+f+\ldots$;

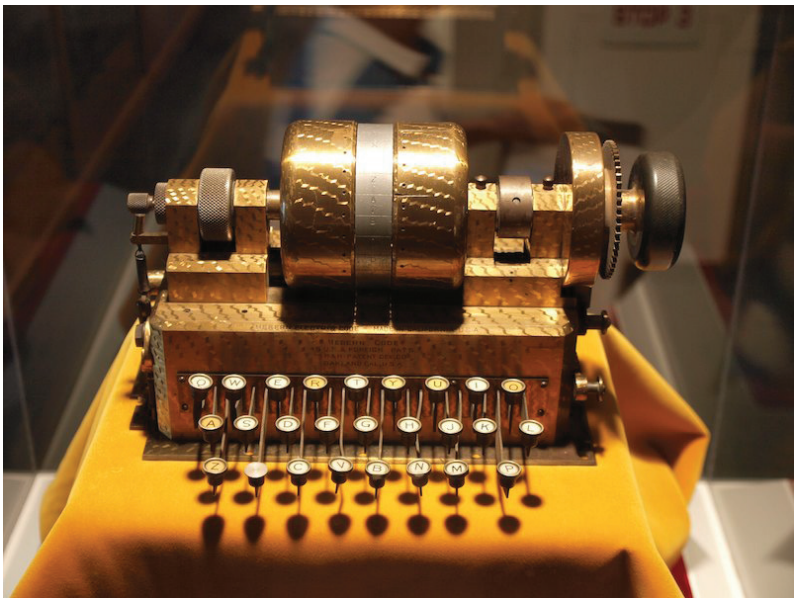
Step 3. It inverts the order of digits in the number n and gets the number $n' = \overline{\ldots fedcba}$;

Step 4. As a result of the encryption the machine prints the number $m = n'+2\cdot S_n$.

But now the cipher machine is broken: sometimes it works correctly but sometimes it prints random numbers m.

After encryption of her secret number n Mary found out that the result is the power of two, $m = 2^k$ for some integer k.

Determine was it the correct encryption in this case?

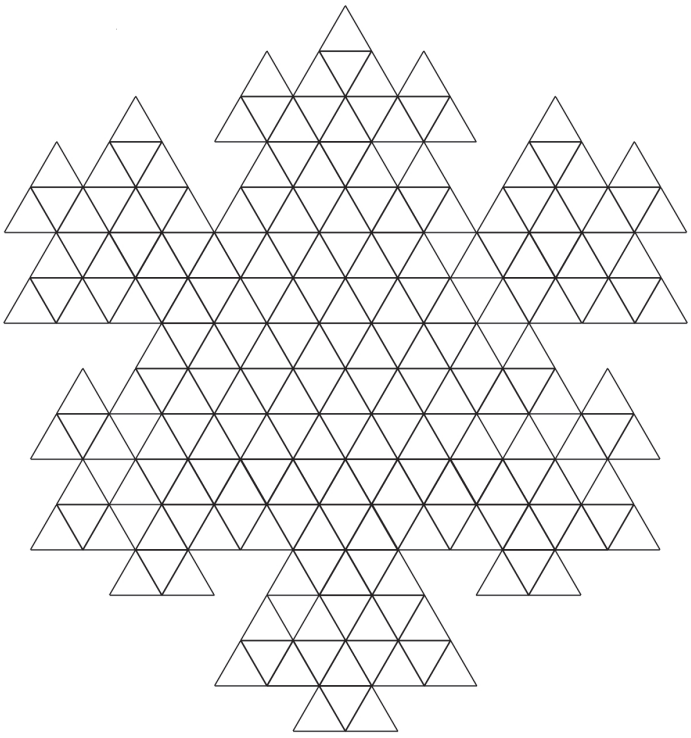


Task 6. «The Snowflake cipher»

Alice wants to encrypt some text using the Snowflake cipher. Encryption is described by the following algorithm:

Step 1. Choose an arbitrary small triangle in the snowflake (see below);
Step 2. Put the first letter of your message into this triangle;
Step 3. Write the next letter of the message (without spaces) into an arbitrary empty neighbouring triangle. Neighbouring means having a common edge. Repeat this step until the end of message.
Step 4. After inserting of all the letters, write down the text from snowflake in horizontal order from top to bottom and left to right.

Determine what is the maximal possible length of a message that can be encrypted with the Snowflake cipher?



An example. We want to encrypt the message: `LOOK HOW IT WORKS`. As a result we can get the ciphertext: `LHOWOOKITSKROW`.